

## SHORTCUTS IN THE FIFTH DIMENSION

Robert Caldwell

*Department of Physics & Astronomy*

*Dartmouth College*

*6127 Wilder Laboratory*

*Hanover, New Hampshire 03755 USA*

David Langlois

*Institut d'Astrophysique de Paris,*

*Centre National de la Recherche Scientifique,*

*98bis Boulevard Arago, 75014 Paris, France*

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### Abstract

If our Universe is a three-brane embedded in a five-dimensional anti-deSitter spacetime, in which matter is confined to the brane and gravity inhabits an infinite bulk space, then the causal propagation of luminous and gravitational signals is in general different. A gravitational signal traveling between two points on the brane can take a “shortcut” through the bulk, and appear quicker than a photon traveling between the same two points along a geodesic on the brane. Similarly, in a given time interval, a gravitational signal can propagate farther than a luminous signal. We quantify this effect, and analyze the impact of these shortcuts through the fifth dimension on cosmology.

### I. INTRODUCTION

The idea that our Universe may be a boundary of a larger spacetime manifold has triggered an outburst of creative and profound research in particle physics and cosmology. The notion of a boundary or brane-world was first made concrete in Horava-Witten theory [1], an M-theory in which the gauge fields are confined to a series of fundamental domain walls and gravity inhabits the bulk space between the walls. Inspired by such M-theory developments, the extra dimensions have been exploited in a variety of situations, notably in an explanation of the mass scale hierarchy problem [2]. The Randall-Sundrum model [3] has demonstrated that extra dimensions need not be compact or even small, leading to fascinating speculation for cosmology and experiment. That is, these extra dimensions are not just the realm of abstract theory, but may have observable consequences ranging from astrophysics [4–6] and accelerators [7–10] to the laboratory [11–13].

The brane-world has become a new forum for the investigation of cosmology. Numerous studies have explored the dynamics of inflation, or the generation and evolution of fluctuation spectra in the early Universe brane-world. A significant result which has spurred on much work is the analog of the FRW equation for the cosmic evolution on a three-brane embedded in five-dimensional anti-deSitter [14,15]. Although the gravity is Einsteinian, the backreaction of the curvature at the brane/bulk interface onto the brane causes the cosmological expansion law to become

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa_{(5)}^4}{36} \rho_{brane}^2 + \frac{\Lambda}{6} \quad (1)$$

where the five-dimensional Newton's constant and mass scale are related by  $\kappa_{(5)}^2 = M_{(5)}^{-3}$  and where the four-dimensional Planck mass is given by  $M_{pl}^2 = M_{(5)}^3 \ell$ ,  $\ell \equiv \sqrt{-6/\Lambda}$  being the anti-deSitter radius of curvature. It has been recognized [16] that if the brane carries, in addition to ordinary matter, a tension  $\sigma$  (such that  $\rho_{brane} = \rho + \sigma$ ) which compensates the effect of the bulk cosmological constant  $\Lambda$ , or more precisely such that  $\kappa_{(5)}^4 \sigma^2 = -6\Lambda$ , the standard expansion law can be recovered in the late-time limit when  $\rho \ll \sigma$ .

Few brane-world studies have considered the simpler, yet deeper issue of causality. In previous work, Chung and Freese [17] have demonstrated that a null geodesic passing through an extra dimension can connect points in the lower-dimension which are causally disconnected with regard to null geodesics confined to the brane. They further speculated that such null geodesics could be used to solve the cosmological horizon problem, in place of inflation. However, they did not consider a realistic spacetime in their analysis. Next, Ishihara [18] has shown quite generally that the condition for the existence of ‘‘causality violating’’ null geodesics which pass through the anti-deSitter bulk is merely the deviation from a pure tension-like stress-energy tensor. That is, provided  $\rho_{brane} + p_{brane} > 0$ , then the extrinsic curvature bends the brane concave towards the bulk, allowing for the existence of such null geodesics. These two results serve as the starting point for our investigation.

In this paper, we reduce the analysis of graviton propagation in an infinite, warped bulk into a practical form. Our principle result is a useful expression for what we will call the ‘‘gravitational horizon radius’’ in contrast to the standard, photon horizon radius in an FRW spacetime. This result will allow us to demonstrate that the horizon problem is not so easily solved: although light is supplanted by the graviton in determining the causal structure of the brane-world, the effect in a realistic scenario is small.

## II. THE SPACETIME

The starting point for our investigation is a five-dimensional spacetime, analogous to the Randall-Sundrum model, where we take the extra dimension to be infinite in extent.

### A. The bulk

In fact, a generalization of the bulk spacetime is Schwarzschild - anti-deSitter, which we find convenient for this analysis. The metric can be written as

$$ds^2 = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2d\Sigma_k^2, \quad (2)$$

where  $d\Sigma_k^2$  stands for the metric of maximally symmetric three-dimensional spaces ( $k = 0$  for a flat three-space,  $k = 1$  for a three-sphere,  $k = -1$  for a hyperbolic three-space), and with

$$f(R) = k + \frac{R^2}{\ell^2} - \frac{\mu}{R^2}. \quad (3)$$

Here,  $\ell$  is the constant curvature radius of anti-deSitter and  $\mu$  is the five-dimensional Schwarzschild-like mass. We will be interested primarily in the simplest case  $k = \mu = 0$ . In addition, we take the bulk to be empty. This is not generally true, as the bulk is typically filled with other fields such as a supergravity multiplet, as well as other branes, in more realistic models. Nevertheless, provided these additional elements are minor, *e.g.* the energy due to the additional fields is negligible compared to the negative cosmological constant, and the additional branes are distant, then our assumption of an empty bulk should be reasonable.

## B. The brane

We assume that the spacetime of the three-brane is homogeneous and isotropic. Therefore, the trajectory of the brane is simply determined by its position in the fifth dimension, *i.e.* by a function  $R_b(T)$ . In other terms, the problem of the motion of the brane in the bulk is analogous to the motion of a particle in a two-dimensional spacetime with coordinates  $R$  and  $T$ .

It is useful to introduce the proper time  $t$  for the brane, defined by

$$dt^2 = f(R_b)dT^2 - \frac{dR_b^2}{f(R_b)}, \quad (4)$$

so that

$$dT = \frac{\sqrt{f(R_b) + \dot{R}_b^2}}{f(R_b)} dt \quad (5)$$

where the dot indicates the derivative with respect to  $t$ . Then, the induced metric on the brane is simply

$$ds_{brane}^2 = -dt^2 + R_b(t)^2 d\Sigma_k^2, \quad (6)$$

and  $R_b(t) \equiv a(t)$  can be identified with the usual cosmological scale factor in the brane-world.

## C. Null geodesics

Our purpose now is to compute the trajectories of null geodesics in the bulk spacetime, which start from some point within the brane. Let us consider such an initial point, A, as illustrated in Figure 1. It is convenient to introduce a spherical coordinate system  $(r, \theta, \phi)$  in

the brane, which is centered on A, so that any signal can be described by a radial geodesic. Then, free to ignore the angular variables  $\theta, \phi$ , we are left with a three-dimensional problem with a metric

$$ds^2 = -f(R)dT^2 + f(R)^{-1}dR^2 + R^2dr^2. \quad (7)$$

To compute the geodesic trajectories, it is convenient to resort to the Killing vectors of the metric, which are here  $\left(\frac{\partial}{\partial T}\right)^a$  and  $\left(\frac{\partial}{\partial r}\right)^a$ . If one denotes  $k^a = (dx^a/d\lambda)$  as the vector tangent to the geodesic, then the existence of these two Killing vectors implies that

$$k_T = -f(R)\frac{dT}{d\lambda} = -E \quad (8)$$

and

$$k_r = R^2\frac{dr}{d\lambda} = P \quad (9)$$

are constants of motion along the geodesics. Imposing moreover that  $k^a$  is a null vector, one finds

$$\left(\frac{dR}{d\lambda}\right)^2 = E^2 - P^2\frac{f(R)}{R^2}. \quad (10)$$

Combining (9) with (10), one easily gets

$$\left(\frac{E^2}{P^2} - \frac{f}{R^2}\right)^{-1/2} \frac{dR}{R^2} = dr. \quad (11)$$

This is the seed of our result, as it relates distances on the three-brane to the radial coordinate in the five-dimensional space or equivalently the expansion scale factor on the brane. In the particular case  $k = \mu = 0$  it is straightforward to integrate to get

$$\frac{1}{R_A} - \frac{1}{R} = \frac{E}{P}\alpha r, \quad (12)$$

with

$$\alpha \equiv \sqrt{1 - \frac{P^2}{E^2\ell^2}}. \quad (13)$$

Similarly, combining (8) with (10), one gets the trajectory of the geodesic along the time coordinate. The infinitesimal version is

$$\frac{dR}{f\sqrt{1 - \frac{P^2 f}{E^2 R^2}}} = dT. \quad (14)$$

Once more, in the case  $k = \mu = 0$  it can be integrated to yield the very simple relation

$$\frac{1}{R_A} - \frac{1}{R} = \frac{\alpha}{\ell^2}(T - T_A). \quad (15)$$

One can also relate directly  $T$  to the radius  $r$ , according to

$$r = \frac{P}{E\ell^2} (T - T_A). \quad (16)$$

Finally, it is possible to get rid of the parameters  $E$  and  $P$  to get the following equation for the geodesic

$$\left(\frac{1}{R_A} - \frac{1}{R}\right)^2 + \frac{r^2}{\ell^2} = \frac{1}{\ell^4} (T - T_A)^2. \quad (17)$$

Let us now denote B as the point where the null geodesic starting from A again crosses the brane. The time difference  $T_B - T_A$  can be expressed, using (5), in terms of the brane proper time, *i.e.* the brane-world cosmic time:

$$T_B - T_A = \ell \int_{t_A}^{t_B} \frac{dt}{a} \sqrt{1 + \ell^2 H^2}. \quad (18)$$

Then we see that between times  $t_A$  and  $t_B$ , the null geodesic has traversed a comoving distance  $r_g$ :

$$r_g = \left( \left[ \int_{t_A}^{t_B} \frac{dt}{a} \sqrt{1 + \ell^2 H^2} \right]^2 - \left[ \int_{t_A}^{t_B} \frac{dt}{a} \ell H \right]^2 \right)^{1/2}. \quad (19)$$

This equation represents the main result of this paper, a simple expression which gives the horizon radius for the causal propagation of gravitational signals between two points on the brane through the bulk. Hence, we call this the *gravitational horizon radius*.

The horizon radius for the causal propagation of luminous signals on the brane, as in the standard FRW cosmology, is given by

$$r_\gamma = \int_{t_A}^{t_B} \frac{dt}{a} \quad (20)$$

where the subscript indicates that this is the path traveled by photons and other fields confined to the brane manifold. We will be interested in cases in which  $r_g$  and  $r_\gamma$  are different. Note that, if our universe was static, *i.e.*  $H = 0$ , which in the present model would correspond to the strict Randall-Sundrum configuration [3], or de Sitter, *i.e.*  $H > 0$  and constant, then the photon horizon and the bulk gravitational horizon would be exactly identical. (This agrees with the results of Ishihara [18], since  $\rho_{brane} + p_{brane} = 0$ .)

### III. CAUSAL DISTANCES

There are two interesting regimes for the evaluation of  $r_g$ , depending on the ratio between the Hubble radius and the five-dimensional length scale. We examine in turn the two regimes.

### A. The low energy regime: $\ell H \ll 1$

This regime corresponds to a universe governed by the standard FRW equation. In this case it is simple to manipulate the integrals in (19,20) using  $dt/a = da/(a^2 H)$  to obtain the ratio of the gravitational to photon distances traveled by a signal propagating between times  $t_A$  and  $t_B$ . Expanding in terms of the small parameter  $\ell H$ , we obtain

$$\begin{aligned} r_g/r_\gamma &\approx 1 + \frac{1}{2} (\ell H_B)^2 \frac{1+3w}{5+3w} \left(\frac{a_B}{a_A}\right)^{(5+3w)/2} \\ &\times \left[ \frac{1 - (a_A/a_B)^{(5+3w)/2}}{1 - (a_A/a_B)^{(1+3w)/2}} - \frac{(1+3w)(5+3w)[1 - (a_A/a_B)]^2}{4[1 - (a_A/a_B)^{(1+3w)/2}]^2} \left(\frac{a_A}{a_B}\right)^{(1+3w)/2} \right] \\ &\sim 1 + \frac{1}{2} (\ell H_B)^2 \frac{1+3w}{5+3w} \left(\frac{a_B}{a_A}\right)^{(5+3w)/2} \end{aligned} \quad (21)$$

where  $w = P/\rho$  is the equation of state of the background matter on the brane (*e.g.*  $w = 1/3, 0$  in the radiation, matter eras), and the last approximation is valid for  $w > -1/3$  and  $a_B \gg a_A$ .

Let us consider a signal which would reach us now, at  $t_B = t_0$ . Then the above ratio reduces to

$$r_g/r_\gamma \approx 1 + \frac{1}{10} (\ell H_0)^2 (1+z)^{5/2}, \quad (22)$$

where  $H_0$  is the present Hubble parameter, and  $z$  the redshift of the source emitting the signal, which we have assumed to be in the matter-dominated era. We see that the magnitude of the time delay depends on the curvature radius,  $\ell$ , of the 5-dimensional anti-deSitter spacetime. However, based on precision tests of the gravitational force law, the size of the extra dimension must be less than  $\sim 1$  mm [13], so that  $\ell H_0 \lesssim 10^{-29}$ . We conclude that, although the time delay increases with the redshift of the source, it is not enough to compensate for the extremely small factor  $(\ell H_0)^2$  in order to obtain a significant cosmological time delay at present.

### B. The high energy regime: $\ell H \gg 1$

This regime corresponds to the early Universe, for energy densities  $\rho \gtrsim \sigma \approx M_{Pl}^2/\ell^2 \approx M_{(5)}^6/M_{Pl}^2$ . Interestingly, the leading contribution to the gravitational distance is independent of  $\ell$ , so that the ratio becomes

$$\frac{r_g}{r_\gamma} \approx \left[ \int_A^B \frac{da}{a^2 H^2} \int_A^B \frac{da}{a^2} \right]^{1/2} / \int_A^B \frac{da}{a^2 H}. \quad (23)$$

At these energy scales the non-standard cosmic evolution of equation (1) applies, with  $H^2 \propto \rho^2$  (*e.g.* radiation, with an equation of state  $w = 1/3$  drives  $H \propto a^{-4}$ ). Therefore we find

$$\frac{r_g}{r_\gamma} \approx \left[ \frac{(2+3w)^2 (a_B/a_A - 1) (1 - (a_A/a_B)^{5+6w})}{(5+6w) (1 - (a_A/a_B)^{2+3w})^2} \right]^{1/2} \sim \frac{2+3w}{\sqrt{5+6w}} \sqrt{\frac{a_B}{a_A}}, \quad (24)$$

where the last approximation is valid for  $w > -2/3$  and  $a_B \gg a_A$ . The ratio  $r_g/r_\gamma$  thus goes to infinity when  $a_A$  goes to zero. However, there is a limit to the applicability of this result, since there is a lower bound on the time for which the physics of this scenario is valid. In standard cosmology, this limiting time is the Planck time. In a model with extra-dimensions, the limiting time is related to the fundamental mass scale of the theory, which is here  $M_{(5)} = (M_{Pl}^2/\ell)^{1/3}$  (with  $M_{(5)} \sim 10^8$  GeV for  $\ell \sim 1$  mm). Indeed, the theory will be invalid for energy densities in the brane higher than  $M_{(5)}^4$ , which corresponds to a cut-off Hubble parameter  $H \sim M_{(5)}$ . As a result of this constraint, the largest ratio for the gravitational to luminous horizon radii is obtained with  $t_B \sim \ell$  and  $t_A \sim M_{(5)}$ , which yields

$$\frac{r_g}{r_\gamma} \sim \sqrt{\frac{a_B}{a_A}} \sim \left(\frac{H_A}{H_B}\right)^{1/8} \sim (M_{(5)}\ell)^{1/8} \sim \left(\frac{M_{Pl}}{M_{(5)}}\right)^{1/4}. \quad (25)$$

where we have assumed a non-standard, radiation-dominated era. With the lowest possible value  $M_{(5)} \sim 10^8$  GeV, this gives a maximum ratio  $r_g/r_\gamma \sim 10^3$ .

Now we turn to the classic horizon problem. The ratio of the horizon radius at the present time  $t_0$  to the horizon radius at some early time  $t_B$  is given by

$$r_{\gamma 0}/r_{\gamma B} = \frac{\int_{t_B}^{t_0} dt/a}{\int_{t_B}^{t_0} dt/a} \approx \frac{a_B}{a_0} \frac{H_B}{H_0}. \quad (26)$$

Since this is a standard textbook problem, it is sufficient to observe that  $r_{\gamma 0}/r_{\gamma B} > 1$  is the essence of the horizon problem. For  $t_B \sim \ell$ , one finds that

$$r_{\gamma 0}/r_{\gamma B} \approx \frac{a_B}{a_0} \frac{H_B}{H_0} \sim [(\ell H_0)^2 (1 + z_{eq})]^{-1/4}. \quad (27)$$

For  $\ell \sim 1$  mm, this gives  $r_{\gamma 0}/r_{\gamma B} \sim 10^{14}$ . It is thus clear that the  $10^3$  ratio between the bulk gravitational horizon and the usual horizon is quite insufficient to account for the horizon problem. Even relaxing the bound on  $\ell$  due to gravitational experiments today (by considering an effective bulk cosmological constant that varies with time so that  $\ell$  contracts on millimeter scales after nucleosynthesis) would not be sufficient. Indeed, the constraint on  $\ell$ , or  $M_{(5)}$ , would then be the nucleosynthesis constraint, which can be expressed by the condition

$$\sigma^{1/4} < 1 \text{ MeV}, \quad (28)$$

implying a minimum mass  $M_{(5)} \sim 10^4$  GeV (since  $\sigma \sim M_{(5)}^6/M_{Pl}^2$ ). The ratio  $r_g/r_\gamma$  given by (25) can then be increased by one order of magnitude up to  $10^4$ , while the ratio  $r_{\gamma 0}/r_{\gamma B}$  can be decreased to the value  $10^8$ . Still, this is not enough to solve the horizon problem.

#### IV. ANALYSIS

We have shown that shortcuts through the fifth dimension, with the gravitational horizon radius given by equation (19), are not short enough to solve the classical horizon problem. We have furthermore argued that a time evolving bulk energy density, which would permit

$\ell$  to start out large and decrease with time, cannot fully solve the problem due to other constraints.

We caution that our results are valid strictly for the case of an empty bulk spacetime with a single, infinite, extra dimension as described in this paper. Motivated by the Randall-Sundrum scenario and other work on brane-world cosmology, this case has added appeal due to the simplicity of the geodesic paths. Naturally, one may ask how these results apply to more general cases. Since the shortcut is a consequence of the warping of space in the extra dimension, there is no shortcut for compact, flat extra dimensions. We have not explored the case of more than one extra warped dimension (we are unaware of any such models in cosmology), though such scenarios, with additional bulk fields, might be more realistic in the context of certain particle physics models. Once the spacetime metric is known, one should repeat the procedure described in this paper.

Returning to our specific results, the difference between the gravitational and photon horizons, may be enough to provide for some very interesting physics. Specifically, gravitational effects on the brane propagate outside the light cone, as illustrated in Figure 2, due to the shortcuts through the fifth dimension, forcing a redefinition of past and future causal domains. The communication of gravitational effects, both radiative and non-radiative degrees of freedom, over length scales  $r_g \gg r_\gamma$  beyond the influence of fields on the brane has not yet been investigated. While we cannot comment decisively, this seems to have an important bearing on at least two problems: the initial conditions for inflation, and phase transitions in the early Universe.

The inflaton must be homogeneous and potential-dominated over a region larger than the horizon volume in order to initiate inflationary expansion. (See [19] for details.) If information about the gradients or inhomogeneities in the inflaton field are carried by gravity, then correlations in the inflaton can arise on length scales  $r_g \gg r_\gamma$ . (Of course, it has already been pointed out that correlations can exist on superhorizon scales [20], but the amplitude must decay [21].) The outcome depends on whether the gravitational interaction leads to dissipation or amplification of inhomogeneities.

The rate at which a phase transition proceeds in the expanding Universe, and the formation of topological defects through the Kibble mechanism hinges on the relative sizes of the correlation length of the order parameter and the causal length scale. If information about the fields involved in the phase transition, such as local fluctuations in the energy density, are carried by gravity, this could affect the rate of the phase transition, and the rate at which topological defects are formed.

We also pause to mention that other analyses of phase transitions and challenges to inflation (specifically, the flatness problem) have been carried out in the context of the brane-world [22,23].

Finally, we note that it is unlikely that shortcuts due to a local gravitational distortion of the brane have an observable effect. Assuming that the ratio  $\ell/\lambda$  plays a similar role as  $\ell H$  in determining the size of the shortcut, where  $\lambda = c(r^3/GM)^{1/2} \sim 10^{13}$  cm for the Earth, then the effect is negligible.



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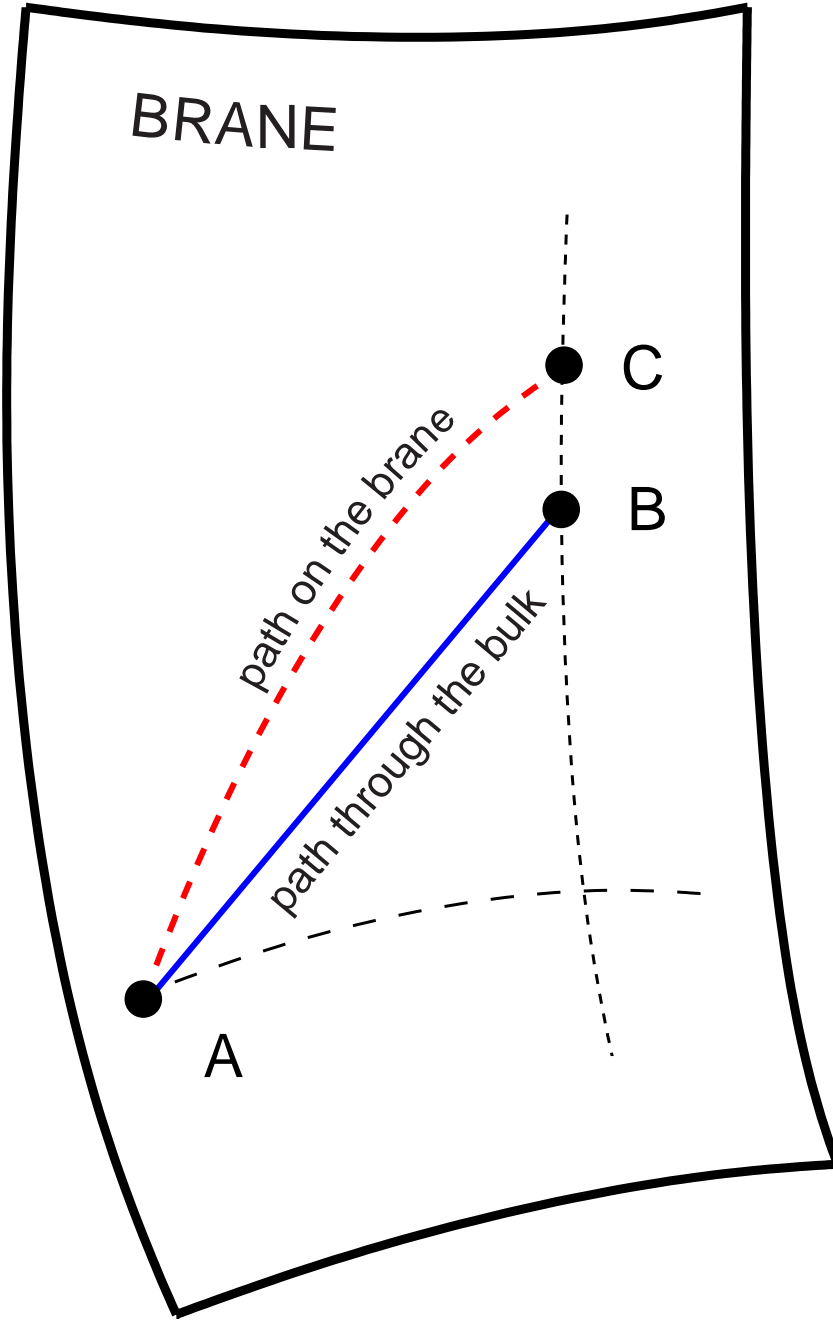


FIG. 1. The null geodesics on the brane and through the bulk are represented schematically. The line passing from points A to C represents a null geodesic on the brane, whereas the points A and B are joined by a null geodesic which takes a shortcut through the bulk. The light, long dashed line passing through A represents a hypersurface of fixed cosmological time on the brane; the light, short dashed line passing through B and C represents the trajectory of points at a fixed comoving position.

gravitational horizon

photon horizon

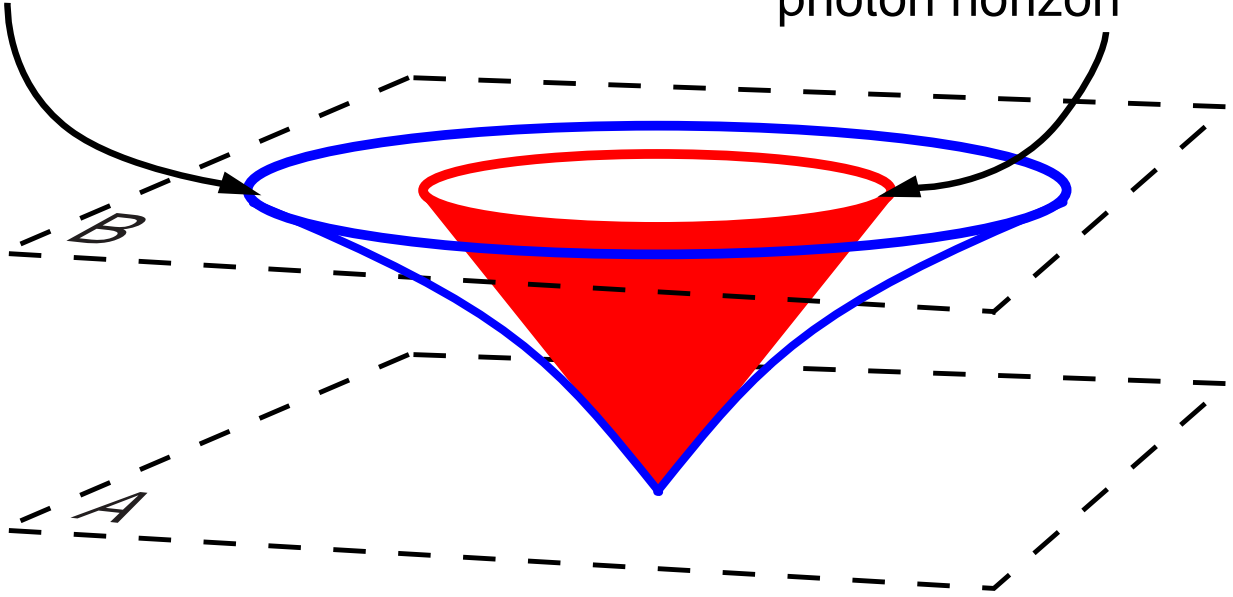


FIG. 2. The difference between the conformal gravitational and photon horizon in the high energy regime ( $\ell H \gg 1$ ), between spatial hypersurfaces at times A and B is illustrated. The future conformal photon horizon grows linearly with conformal time, whereas the conformal gravitational horizon grow as a power law. By flipping the diagram upside down, we can see that the past gravitational horizon becomes larger for earlier starting time. However, as argued in the text, the effect is not enough to solve the horizon problem within the constraints of the five-dimensional theory.

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